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Hw3 analysis

***Question 1***

Considering a simple 1st order chemical reaction, substance A becomes B with rate constant k1, and B becomes C with rate constant k2. From mass conservation principles the governing equation for describing the time rate of change of the concentration for substance A, B and C can be expressed as

Find the transient solution of the reaction system for the given initial conditions:

***Answer***

Write equation in matrix nation

Which yields 3 eigenvalues

and 3 eigen vectors

For

This is equivalent to 3 equations

Canceling terms

I have the work done by hand but I did not type it up but it is done using the same technique

For

This is equivalent to 3 equations

Canceling terms

For

This is equivalent to 3 equations

Canceling terms

From eq2, substitute into x\_2 to get unknown value

The general solution is

Using initial conditions

From eq1 we get

From eq2 we get

From eq3 we get

The following were helpful links for the problem:

"mathematics for physical chemistry" p256 [books.google.com](http://books.google.com)

<http://books.google.com/books?id=QL0GXO4CeawC&lpg=PA256&ots=NL3J5xJFfY&dq=help%20what%20is%20the%20solution%20dA%2Fdt%20%3D%20-k1%20A%20%20%20%20%20%20dB%2Fdt%20%3D%20k1A%20-%20k2B%20dC%2Fdt%20%3D%20k2B&pg=PA256#v=onepage&q&f=false>

also

<http://www.boards.ie/vbulletin/showthread.php?p=70061453>

***Question 2:***

Find the general solution of the 3rd order ODE:

***Solution:***

First need the solutions of the homogeneous equation

That is a linear constant coefficient ODE with characteristic equation

It has a real root at 0

Hence

Must be one term in the complementary function

When the

And two conjugate complex roots

Thus the complementary function must also contain

Solution of the homogeneous equation is

If y1 y2 and y3 are solution of the homogeneous equation

The wronskian is

Note: For this type of problem the particular solution is the whole solution, you don’t need to and the homogeneous solution to the particular like the lower order ode

Variation of parameters methods assumes that general solution of the inhomogeneous equation takes the form:

The functions v1 v2 and v3 can be found by the integrals

W is the wronskian of the fundamental system. Wi is the same determinant as W, but column I is replaced by < 0, 0 ,f(x) > f(x) is the function on the RHS of the inhomgeneous equation

For the given equation

For W1

Note: sec x = 1/(cos x) and tan x = (sin x)/(cos x)

For W2

For W3

For v1

For v2

Set

Then we find

Substitute du = -sin x, u = cos x

For v3

Hence the solution is

Helpful links

http://en.wikipedia.org/wiki/Euler%27s\_formula

Example 1 Pg103 “Advanced engineering mathematics” 2nd edition

Also pg80 for wronskian (pg 83 for example 1 euler chauchy)

http://au.answers.yahoo.com/question/index?qid=20110221094122AAsWBNO

http://answers.yahoo.com/question/index?qid=20080901053216AA5k9KZ

***Question 3***

The following Poisson equation is often used for describing a screened electric potential around a changed spherical particles in an liquid electrolyte

Where r is the distance away from the center of the particle, a is the particle radius, phi is the local electic potential, and k controls the screening effect. If k=0, no screening and the equation becomes 1d laplace equation. Solve for the local electric potential and compare your results for various k values

***Answer***

denotes divergence

The solution of this second-order differential equation must satisfy the following first-order differential equation:

This differential equation can be rewritten as

Separation of variables

To compare different k values just plug this equation into matlab and evaluate and plot

Useful link:

<http://hyperphysics.phy-astr.gsu.edu/hbase/electric/laplace.html>

<http://en.wikipedia.org/wiki/Electric_potential>

<http://mathworld.wolfram.com/SphericalCoordinates.html>

http://teacher.pas.rochester.edu/PHY217/LectureNotes/Chapter3/LectureNotesChapter3.html

http://www.phengkimving.com/calc\_of\_one\_real\_var/16\_diffl\_eq/16\_01\_first\_ord\_eq/16\_01\_02\_var\_sep\_eq.htm

***Question 4***

Find the general solution of

***Answer***

Sounds like we can use a sum of infinite series method, because you said “near”, which we did in class

First we note that the equation does not have constant coefficients. Next we see that x =0 is an ordinary point of the equation. Existence and uniqueness theory guarantees that this IVP has a unique solution.

There is a solution of the form

Substituting in the differential equation we get

Note:

therefore

This equation is true if the coefficient of x” is 0:

We solve this recursion relation by putting n=0,1,2,3 successively in equation 1

Put n=0:

Put n=1:

Put n=2:

Put n=3:

The solution is

Helpful link

P1199 stewart calculus

Also class notes on 9/21/2011 pg4-5

<http://calculusplus.cuny.edu/ODE_Series%20Solutions1.html>

***Question 5***

Find the general solution of the ODE

***Answer***

X=0 is a regular singular point

Let

Substituting to ODE eq we obtain

For our a’s we get:

Therefore

***Question 6***

Show that the differential equation

Has an irregular singular point at x=0. Show that the simple transformation  
 can transform the equation to

With a regular singular point at t=0

***Answer***

Transform the equation

Because

Because of this the function is not analytic at the point x=0

This means that the ode has an irregular singular point at x=0

Now we need to transform the equation by

In conclusion

As said in the problem statement the transformed equation has a regular singular point at t=0

Useful link:

Redo this problem after looking at pg405 of advanced engineering math 2nd edition

http://www.physicsforums.com/archive/index.php/t-321945.html

<http://en.wikipedia.org/wiki/Analytic_function>

http://mathworld.wolfram.com/MaclaurinSeries.html

<http://www.mathcs.citadel.edu/~chenm/335.dir/03fal.dir/lect5_2.pdf> (recurence relations)

Chapter 7.7 in haberman

Pg 6 in series solution handout